Welfare in a Unionized Bertrand Duopoly

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Abstract
This paper presents a model of unionized oligopoly under Bertrand competition to investigate the effects of unionization parameters on domestic welfare. We find that the magnitude of the effect of foreign subsidization on the domestic wage (i.e., the “wage effect”) is critical in determining the direction of the domestic welfare effect. If a unionization parameter raises the absolute value of the wage effect then domestic welfare is positively associated with that unionization parameter. We apply our findings to the case of profit sharing and find that while profit sharing raises domestic welfare under free trade, it paradoxically reduces welfare in a policy equilibrium.

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1. Introduction

The area of optimal trade policy in imperfectly competitive markets has been widely researched in recent years. Since large oligopolistic firms are often unionized, researchers have also focused on the analysis of unionized oligopolies. Brander and Spencer (1988) focus on trade policy under unionization for the cases of both export and import competition. Fung (1989 and 1995) investigates the role of profit sharing between the union and the firm on market share rivalry. Mezzetti and Dinopoulos (1991) analyze efficient contracting between a domestic firm and the union and its effects on import competition. Gaston and Trefler (1995) explore the effects of trade policy on union wages. Recent contributions of Santoni (1996) and Zhao (1995 and 1998) have focused on issues relating to sequential bargaining and foreign direct investment, respectively.

Tanaka (1994) finds a rather remarkable result that in an otherwise level playing field the role of profit sharing is to reduce market share at a subsidy equilibrium. Bandyopadhyay and Bandyopadhyay (1999, henceforth B&B) find a similar paradox in a standard unionized oligopoly model where rival governments (simultaneously) engage in profit shifting policies. In an otherwise symmetric model the unionized firm has a larger market share than its non-unionized rival. Recently, Bandyopadhyay and Bandyopadhyay (2000b) show that paradoxical welfare results emerge under unionization for both an efficient bargaining model and a right to manage model under Cournot competition.

Bandyopadhyay and Bandyopadhyay (2000a) analyze a unionized Bertrand duopoly and show that unlike the non-union case the optimal policy does not revert from a subsidy to a tax
It is well known that the strategic trade policy is not robust to the mode of competition in the non-union case. In the light of these contributions a natural question is whether the welfare results are also robust to the mode of competition under unionization. We address this issue by providing a welfare analysis for a unionized Bertrand duopoly.

We do not specify a particular model of union behavior and can accommodate wage-bargaining and profit sharing as special cases. The only restriction that we impose is that we have a three stage model like Brander-Spencer (1988) where the export subsidy, union wage and employment are chosen in stages one, two and three, respectively. We use a third country export rivalry model. The home firm is unionized while its foreign rival is non-unionized.

A tax by the foreign nation under (differentiated good) Bertrand competition raises the foreign firm’s price. This leads to a larger demand for the home firm’s product and raises the demand for labor. In the normal case, this will raise the union wage (referred to as the “wage effect” hereafter). Thus, the home firm’s price is affected in two ways due to the rise in the foreign price. First, the home price will rise as a Bertrand reaction. In addition, the rise in the union wage raises the marginal cost of the home firm, thereby raising its price further. The “wage effect” amplifies the domestic price reaction and causes the foreign nation to tax more aggressively. A larger foreign tax causes a positive spillover on the domestic nation, thereby raising its welfare. Thus, a rise in any unionization parameter that amplifies the “wage effect”

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1It is well known that the strategic trade policy is not robust to the mode of competition in the non-union case. A strategic export subsidy obtains in the Cournot case (Brander and Spencer, 1985), while a tax is optimal under Bertrand competition (Eaton and Grossman, 1986).
must raise domestic welfare.

We should note that the results hinge on the foreign government’s ability to gauge the effect of its subsidy (or tax) on the domestic wage. Admittedly, the information requirement of the foreign government regarding the precise model of union behavior is quite demanding. In the same token, however, it is unreasonable for the foreign government to ignore the endogeneity of the union wage. A weaker and a reasonable interpretation of our results is that while the foreign government may not know the precise magnitude of the wage cut, it will recognize the negative relation between the domestic wage and employment and therefore will subsidize less aggressively (or tax more aggressively in the Bertrand case). Compared to the non-union case, this will imply that domestic welfare will be higher under unionization and will increase with the foreign government’s expectation regarding the union “wage effect”.

2. The Model

A home firm (1) is assumed to compete with a foreign firm (2) in a third country market. The firms produce differentiated products $q^1$ and $q^2$, respectively. There is no domestic consumption of these goods in nations 1 and 2, where these firms belong. Each firm is assumed to maximize profits and employ the Nash- Bertrand assumption regarding its rival’s price level. $q^1$ and $q^2$ are substitutes and follow the direct demand functions:

$$q^i = q^i(p^i,p^2), \text{ and, } q^j = q^j(p^i,p^2); \quad q_i < 0 \text{ and } q_j > 0, \quad i,j=1,2; \quad i \neq j.$$  \hspace{1cm} (1a)

The production functions are:

$$q^i = L^i, \text{ and, } q^j = L^j.$$  \hspace{1cm} (1b)
Let \( \alpha \) (0 \# \alpha < 1) be the share of profit given to the domestic union.\(^2\) Let the domestic and the foreign governments offer subsidies \( s \) and \( s^* \), respectively to their firms. The net profit of firm-\( i \) is:

\[
\delta^{1N} = (1-\alpha)\delta^l = (1-\alpha)(p^1 - w + s)q^1(p^1, p^2); \quad \text{and,} \quad \delta^2 = (p^2 - b + s^*)q^2(p^1, p^2).
\] (2)

The home firm’s profit maximization yields the following Nash-Bertrand reaction function:

\[
(p^1 - w + s)q^1 + q^l = 0 \quad p^l = p^l(p^2, w, s);
\] (3)

The foreign firm’s Bertrand reaction function is:

\[
(p^2 - b + s^*)q^2 + q^2 = 0 \quad p^2 = p^2(p^1, s^*).
\] (4)

The third stage Bertrand equilibrium is:

\[
p^l = p^{LB}(w, s, s^*) \quad \text{and} \quad p^2 = p^{LB}(w, s, s^*), s^*) = p^{2B}(w, s, s^*).
\] (5)

The union wage is chosen in the second stage of the game and internalizes (5) in its choice.\(^3\) Therefore, it is a function of the variables chosen in the first stage of the game and of the unionization parameter \( \alpha \) (which for example may be the union’s bargaining power in the Nash bargain function or the degree of profit sharing, \( \alpha \), etc.):

\[
w = w^B(s, s^*, \alpha).
\] (6)

Let the domestic alternative wage be equal to the foreign wage \( b \). This equalizes the social opportunity cost of labor in the two exporting nations and neutralizes the de Meza (1986) and \( \ldots \)

\(^2\)Note that \( \alpha \) can be zero without any change in the following analysis. This model includes but is not limited to the case of profit sharing.

\(^3\)There is no need here to describe the process of that choice. It could be chosen unilaterally by the union as in a monopoly union model to maximize utility, through bargaining between the union and the firm as in Brander and Spencer (1988) or by a profit sharing union. This framework is general and includes all these possibilities.
Neary (1994) type effects. Domestic welfare $W$ is the rent earned from exporting to the third nation. It is also measured by adding the rents earned and the tax burdens of the different agents in this model:

$$W = \delta^{IN} + \delta^{d} + (w-b)q^{i} - sq^{f} = (p^{i} - b)q^{i} = (p^{i} - b)q^{i}(p^{i},p^{2}(p^{i},s^{*})).$$ (8)

This completes the description of the model structure. The free trade and the policy equilibrium can be obtained using this structure.

### 2.1 Welfare under free trade ($s=s^{*}=0$) :

Using (4), (5) and (8), under free trade:

$$dW/d\alpha = \{(p^{i}-b)(q^{i}_{1} + q^{i}_{2}p^{2}_{1}) + q^{i}\}(dp^{i}/d\alpha)$$

$$= \{(p^{i}-b)(q^{i}_{1} + q^{i}_{2}p^{2}_{1}) + q^{i}\}p^{1B}(dw/d\alpha).$$ (9)

Using (3), (9) can be written as:

$$dW/d\alpha = \{(w-b)q^{i}_{1} + (p^{i}-b)q^{i}_{2}p^{2}_{1}\}p^{1B}(dw/d\alpha).$$ (10)

The first term inside the curly bracket reflects the labor market distortion and the second term the strategic distortion. They are of opposite signs and \textit{a-priori} the sign of the bracketed term cannot be determined. However, B&B (2000a) show that the labor market distortion dominates in a wide range of cases. Let us assume that to be the case for our purpose.\footnote{There is no loss of generality, if the sign is opposite the welfare effect in the free trade case is reversed. However, for the profit sharing example that we consider, it can be shown that the labor market distortion dominates as in B&B. The welfare effect for the subsidy equilibrium is independent of this issue.} Thus, the bracketed term is negative. It is easy to check using the first order conditions of the two firms that $(p^{1B}_{w})$ is positive. Thus, we can state the following proposition.
Proposition-1

Domestic welfare must rise if the domestic wage is reduced by a rise in a labor market parameter.

Proof and Comment:
The proof is contained in (10) and the discussion following it. If the domestic wage is raised it raises \( p^1 \). This has two effects. First, \( p^2 \) rises as a Bertrand reaction and causes a positive spillover for firm-1. On the other hand, because of unionization, \( p^1 \) is too high (due to the wage distortion). A further rise in \( w \) will raise \( p^1 \) further, increasing the distortion. As long as the wage distortion effect dominates, welfare will rise (fall) as \( w \) falls (rises). \( \bar{A} \)

Corollary-1

Under demand linearity and a rent maximizing union utility function, a rise in the degree of profit sharing by the union must raise domestic welfare under free trade.

Proof and Comment:
Let us first state the wage determination problem for the general case where \( s \) and \( s^* \) can assume any value.\(^5\) We will then obtain the free trade wage as a special case. Let the union objective function be:

\[
U = (w-b)q^1 + \alpha(p^1-w+s)q^1. 
\] (11)

The union’s objective is to maximize the sum of rents and its share of profit (\( \alpha \) is the degree of

\(^5\)Thus, we can use the union wage from (12) below for analyzing the subsidy equilibrium for profit sharing (presented later in Corollary-2).
profit sharing). The wage is chosen by the union in the second stage of the game. To ensure subgame perfection the union must consider the effect of w on the third stage choice variables \((p^1, p^2)\). In effect the union maximizes (11) subject to (5) above. Assume (for this corollary) that demand takes the form:

\[
q^1 = a - p^1 + ep^2, \quad and, \quad q^2 = a - p^2 + ep^1, \quad 0 < e < 1.
\]  

Using (1c) and (11), the first order condition for maximization of w yields:

\[
w^B(s,s^*,\alpha) = \frac{b(e^2-2) - B^*C}{X}, \quad where, \quad B^* = 4-e^2 + 2\alpha(e^2-2);
\]

\[
C = a + \frac{(a-s)(e^2-2) + e(a+b-s^*)}{(4-e^2)}; \quad X = 2(e^2-2)(4-e^2+\alpha(e^2-2))/(4-e^2).
\]  

Under free trade, (12) can be used to establish that \((dw/d\alpha)\) is negative. Further, it is easy to check that the product of the remaining terms on the right hand side of (10) is negative. Thus, using (10) it is clear that domestic welfare \(W\) will rise with the degree of profit sharing (i.e., \(\alpha\)).

\[\bar{\lambda}\]

2.2 Welfare at the policy equilibrium:

Using (8) we get:

\[
dW = \{(p^1-b)(q_1^1 + q_2^1p_1^2) + q^1\}dp^1 + (p^1-b)q_2^1p_2^2ds^*.
\]  

Under optimal subsidization (or taxation), where \(s^*\) is taken as given by the home government, the marginal effect of \(p^1\) on \(W\) will be neutralized. Therefore, using (3) and (13):

\[
(p^1-b)(q_1^1 + q_2^1p_1^2) + q^1 = 0 \quad s = w-b + \{(p^1-b)q_2^1p_2^2/q_1^1\}.
\]  

This is the optimal subsidy formula derived in B&B (2000a). As pointed out in that paper the subsidy is likely to be positive for several demand and unionization specifications, but the possibility of a tax cannot be ruled out. It turns out that the welfare results for the policy
equilibrium that we derive below are independent of the sign of the trade policy. Using (13) and (14):

\[ \frac{dW}{da} = (p^1 - b)q_1p_1^2(s^*/da). \]  

(15)

\( q_1 \) is positive and using (4) it is clear that \( p_1^2 \) is negative. Therefore, the sign of \( \frac{dW}{da} \) is the negative of \( \frac{ds^*/da}{da} \). The optimal subsidy rule for the domestic government implicitly defines:

\[ s = s(s^*, a^*). \]  

(16)

Define:

\[ w^B_s(s, s^*, a^*) = \frac{Mw^B}{Ms^*}. \]

Evaluate \( w^B_s(s, s^*, a^*) \) at the optimal \( s \) given by (16):

\[ w^B_s(s(s^*, a^*), s^*, a^*) = \mu^B(s^*, a^*). \]  

(17)

It can be shown that the sign of \( (ds^*/da) \) is the same as the sign of \( (M\mu^B/Ma^*) \).\(^6\) Therefore, using (15) we can conclude:

\[ \frac{dW}{da} \geq 0 \quad \text{as} \quad M\mu^B/Ma^* \geq 0. \]  

(18)

Proposition-2

At the policy equilibrium, if a rise in the domestic labor market parameter (\( a \)) raises (reduces) the absolute value of the wage effect (i.e., \(-\mu^B\)), domestic welfare rises (falls) with a rise in \( a \).

Proof and Comment:

The proof relies on (15) through (18) and the mathematical appendix at the end. A tax by the

\(^6\)A sketch of the proof is provided in the appendix.
foreign government raises $p^2$ and increases the demand for $q^1$.\(^7\) $p^1$ rises as a Bertrand reaction.

However, the rise in the demand for domestic labor leads to a rise in the union wage and thereby raises the (net) marginal cost of the domestic firm. This leads to a further rise in $p^1$.

Thus, the Bertrand reaction of $p^1$ to a change in $p^2$ is amplified by domestic unionization.

Consequently, the strategic tax which is set to exploit the Bertrand reaction will be larger. The greater the magnitude of the wage change the greater will be the strategic foreign tax (i.e., lower will be the foreign subsidy). A tax on the foreign firm which raises its price, raises the demand faced by the domestic firm. This raises $q^1$ and consequently domestic welfare. $\bar{A}$

**Corollary-2**

At the policy equilibrium (under demand linearity and a rent maximizing union utility function) a rise in the degree of profit sharing by the union must reduce domestic welfare.

**Proof and Comment:**

Use the same specific functional forms for union utility and the demand curves as for Corollary-1 (i.e., equations 1c, 11 and 12). Using (12):

$$w^*_s = -(B/X)(M/M^*) = B^e/[2(e^2-2)(4-e^2 + d(e^2-2))] = B^e.$$  \hspace{1cm} (19)

It is easy to check:

$$d(-w^*_s)/d\bar{a} = d(-\mu^B)/d\bar{a} < 0. \hspace{1cm} (20)$$

\(^7\)The appendix presents the optimal foreign subsidy. If $w^*_s$ is negative then this subsidy is negative. If $s^*$ rises it reduces the foreign price and thereby reduces the labor demand facing the domestic union. This demand reduction would normally translate to a wage cut. Thus, one would expect $w^*_s$ to be negative. Therefore, the optimal foreign policy is a *modified* Eaton-Grossman (1986) type tax.
(18) and (20) imply:

\[ \frac{dW}{d\alpha} < 0. \tag{21} \]

A rise in the degree of profit sharing makes the union more conservative in its response to demand shifts. Foreign taxation raises the demand for labor facing the domestic union. For high degrees of profit sharing the wage response (rise) is small. Therefore, the response of \( p_1 \) is small. The foreign tax is positively related to the response of \( p_1 \). Thus, the foreign strategic tax is small. As the magnitude of foreign taxation declines with profit sharing, the welfare of the domestic nation must fall.

This result paradoxically reverses corollary-1. It is intuitive that profit sharing should help as in Fung (1989). While this is true under free trade, that is not the case at the policy equilibrium of a Bertrand duopoly. This is similar to the paradoxical findings of Tanaka (1994) and B&B (1999 and 2000b) obtained under Cournot competition. Å

3. Conclusion

This paper makes two contributions. To our knowledge it is the first paper to present a systematic welfare analysis for a unionized Bertrand duopoly. Secondly, it finds that paradoxical welfare results obtain that are similar to the case of Cournot competition (analyzed in the existing literature). Thus, we establish the robustness of welfare results for a unionized oligopoly to the mode of competition. Finally, we note that while the paper highlights the case of profit sharing as an example, the propositions are more general and apply to any wage determination process (assuming that the same three stage Bertrand structure is used).
References


Appendix

Here we provide a sketch of the proof supporting footnote-6 and relation-(18). This method is based on an envelope function approach that is developed in B&B (2000b) for the Cournot case. We also borrow from B&B (1999) in deriving the optimal policy rules. Let the welfare function for the foreign government be:

\[ W^* = (p^2 - b)q^2(). \]  \hspace{1cm} (A1)

Using (1a):

\[ dW^* = (p^2 - b)q^2 dp^1 + ((p^2 - b)q^2 + q^2) dp^2. \]  \hspace{1cm} (A2)

Using (5) and (6):

\[ p^1 = p^1B[w^B(s,s^*,\bar{\alpha}),s,s^*] = H^B(s,s^*,\bar{\alpha}); \quad p^2 = p^2B[w^B(s,s^*,\bar{\alpha}),s,s^*] = F^B(s,s^*,\bar{\alpha}). \]  \hspace{1cm} (A3)

Using (A2) and (A3):

\[ MW^*/M^* = (p^2 - b)q^2 H^B(.) + ((p^2 - b)q^2 + q^2) F^B = 0. \]  \hspace{1cm} (A4)

(A4) reduces to the following foreign optimal subsidy (tax) rule:

\[ (p^2 - b)q^2 + q^2 + (p^2 - b)q^2 \bar{n}^B = 0, \text{ where, } \bar{n}^B = H^B(.)/F^B(.) = \bar{n}^B(s,s^*,\bar{\alpha}). \]  \hspace{1cm} (A5)

(4) and (A5) yields the following foreign subsidy formula:

\[ s^* = (p^2 - b)q^2 \bar{n}^B/q^2. \]  \hspace{1cm} (A6)
From (A3):

\[ H_B^2 = (p_{wB}^1)w_{s^*} + p_{s^*}^{1B} ; \text{ and, } F_B^2 = (p_{wB}^2)w_{s^*} + p_s^{2B} . \]  

(A7)

Differentiating the firm first order conditions it can be shown that \( p_{wB}^1 > 0, p_{s^*}^{1B} < 0, p_{wB}^2 > 0, p_s^{2B} < 0 \).

Thus, if \( w_{s^*}^B \) is negative, \( H_B^2 < 0 \), and, \( F_B^2 < 0 \). Thus, \( n^B \) is positive and the optimal foreign policy is to tax if \( w_{s^*}^B \) is negative. While no general presumption can be made, it is intuitive that \( w_{s^*}^B \) is negative as explained in the text. Now the optimal subsidy rule for the domestic government (14) is a function of \( p^1 \) and \( p^2 \) only (given relation 1a). Now, (4) implies that \( p^2 \) is a function of \( (p^1, s^*) \). Thus, (14) must implicitly define target prices for the domestic government:

\[ p^1 = p_1^1(s^*) \quad p^2 = p^2(p^1(s^*), s^*) = p_2^2(s^*) . \]  

(A8)

Let us consider the foreign government’s optimal policy rule assuming that the home policy is chosen optimally. Both policy rules are satisfied at any policy equilibrium and therefore the following function may be used for comparative statics. Also, (14) implicitly defines \( s(s^*, \tilde{\alpha}) \).

Thus, (1a), (14), (A5) and (A8) imply:

\[ \{p_2^2(s^*) - b\}q_2^2(s^*) + q^2(s^*) + \{p_1^2(s^*) - b\}q_1^2(s^*)n^B[s(s^*, \tilde{\alpha}), s^*, \tilde{\alpha}] = 0 \]

\[ Y \cdot \dot{\alpha}^B(s^*, \tilde{\alpha}) = 0. \]  

(A9)

(A9) implies:

\[ -d\beta^* / d\tilde{\alpha} = (M\beta^B/M\tilde{\alpha})/(M\beta^B/M\beta^*) = \dot{\beta}^B_{\beta^*}. \]  

(A10)

Recall from (A7):

\[ H_B^2 = (p_{wB}^1)w_{s^*} + p_{s^*}^{1B} ; \text{ and, } F_B^2 = (p_{wB}^2)w_{s^*} + p_s^{2B} . \]

By totally differentiating the foreign and domestic firm’s first order conditions (and evaluating the derivatives at the profit maximizing prices) we can show that \( p_{wB}^1, p_{s^*}^{1B}, p_{wB}^2 \) and \( p_s^{2B} \) are all
functions of \((p^1, p^2)\) alone. Thus, when \(s\) is optimally chosen, using (A8) we can infer that they are functions of \(s^*\) alone. Thus:

\[
\bar{n}^B = HZ^B/Z = (p^1 w^B(s^*) + p^2 w^B(s^*) + p^2 w^B(s^*) + p^2 w^B(s^*))/\{p^1 w^B(s^*) + p^2 w^B(s^*)\}. \tag{A11}
\]

Now,

\[
\bar{o}^B_{\bar{a}} = (p^2(s^* - b)q^-B(s^*)/[\bar{M}^B(s(s^*, \tilde{a}), s^*, \tilde{a})/\bar{M}]). \tag{A12}
\]

Using (A11):

\[
\bar{n}^B(s(s^*, \tilde{a}), s^*, \tilde{a}) = \{p^1 w^B(s^*) + p^2 w^B(s^*) + p^2 w^B(s^*) + p^2 w^B(s^*)\};
\]

where, as defined in (17):

\[
\mu^B(s^*, \tilde{a}) = w^B(s^*, \tilde{a}), s^*, \tilde{a}). \tag{A13}
\]

Using (A13) it can be shown that the sign of \([\bar{M}^B(s(s^*, \tilde{a}), s^*, \tilde{a})/\bar{M}]\) is the same as the sign of \(-\bar{M}^B/\bar{M}\). Thus, the sign of \(\bar{o}^B_{\bar{a}}\) is also the same as the sign of \(-\bar{M}^B/\bar{M}\). Now (A4) yields:

\[
MW^B/\bar{M}^B = W^B = F^B_2((p^2 - b)q^-B + q^2 + (p^2 - b)q^-B)\bar{n}B = F^B_2(s(s^*, \tilde{a})) = 0. \tag{A14}
\]

The second order condition of foreign welfare maximization is:

\[
W^B_{s^*} = F^B_2(.)Z_{s^*} + ZF^B_{s^*} < 0.
\]

Evaluated at the welfare optimum (i.e., \(Z=0\)), this implies:

\[
W^B_{s^*} = F^B_2(.)Z_{s^*} < 0. \tag{A15}
\]

Since \(F^B_2(.)\) is negative, (A15) implies that:

\[
Z_{s^*} > 0. \tag{A16}
\]

Note that: \(Z(s, s^*, \tilde{a}) = ((p^2 - b)q^-B + q^2 + (p^2 - b)q^-B)\bar{n}B\). From (A9) we can see that \(\bar{o}(.)\) is the same as the \(Z(.)\) function evaluated at the optimal domestic subsidy rate. Thus, we note the following identity:
\[ \dot{\bar{\sigma}}_s(s^*, \bar{\alpha}) = Z(s(s^*, \bar{\alpha}), s^*, \bar{\alpha}) Y \dot{\bar{\sigma}}_{s^*} = Z_s(\mathcal{M}/\mathcal{M}^*) + Z_{s^*}. \] (A17)

Now, \( Z=0 \), implicitly defines the optimal foreign subsidy \( s^*(s, \bar{\alpha}) \). Thus:

\[ s^*_s(s, \bar{\alpha}) = -Z_s/s_s \quad \text{and} \quad Z_s = -(s^*_s)Z_{s^*}. \] (A18)

(A17) and (A18) imply that:

\[ \dot{\bar{\sigma}}_{s^*} = Z_s(1 - s_s^* s_s^*). \] (A19)

Now, from the standard policy reaction function stability condition, we have: \( 1 - s_s^* s_s^* > 0 \).

Hence, using (A16) and (A19) we have \( \dot{\bar{\sigma}}_{s^*} > 0 \). Thus, (A10) ensures that the sign of \( (ds^*/d\bar{\alpha}) \) is the same as the sign of \( (\dot{\bar{\sigma}}_{s^*}) \). Recall from (A12) and (A13) that the sign of \( (\dot{\bar{\sigma}}_{s^*}) \) is the same as the sign of \( \mu^B_{\bar{\alpha}} \). Thus, the sign of \( (ds^*/d\bar{\alpha}) \) is the same as the sign of \( \mu^B_{\bar{\alpha}} \). Using (15) we know that the sign of \( dW/d\bar{\alpha} \) is the same as the sign of \( (\mu^B_{\bar{\alpha}}) \). That is, the sign of \( dW/d\bar{\alpha} \) is the same as the sign of \( (\mu^B_{\bar{\alpha}}) \). This yields relation (18) of the text.