Golden Rules for Wages

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Abstract: We consider a decentralized version of the neoclassical growth model where labor share is chosen by workers to maximize their long run (permanent) wages. In this framework, if the labor share increases relative to the competitive share, workers capture a larger share of a smaller total income in the steady-state. This is because the incentives to invest are lower and the steady-state capital to labor ratio is lower. We find that the “Golden Rule” labor share is equal to the elasticity of output with respect to labor. This is precisely what would obtain under the assumption of competitive factor markets. We also consider the model with two classes of workers: organized and unorganized. In this case, organized labor may chooses a higher than competitive share and the difference is economically significant for plausible parameter values. Furthermore, relative to the Cobb-Douglas case, organized labor chooses a higher share for the empirically relevant case of an elasticity of substitution less than unity. We also analyze a multi-sector version of the model where workers in each sector are organized and choose their share of that sector’s output. The golden rule of wages still holds: each sector’s workers can do no better than to choose the competitive labor share. In summary: organized labor can only improve its lot at the expense of unorganized labor; not at the expense of capital.

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1. Introduction

Consider a world where the labor share of income is not directly related to technological parameters. Suppose instead that labor share depends solely on the bargaining power of workers. If workers in such a world maximize their long-run (permanent) wages, what labor share will they choose? What would be optimal if labor could choose collectively as a class?

Union membership in the US fallen steadily since the 1950s, from around 35 percent down to less than 10 percent (Hirsch, 2008). Defenders of collective bargaining as benefiting labor have found themselves increasingly embattled. Stepping down as chair of the National Labor Relations Board (NLRB) – and in response federal legislation introduced to block specific NLRB actions – Wilma Leibman lamented the “perception of [the] agency as doing radical things”:

Some say collective bargaining is antithetical to the economy[. ...] I don’t buy that at all. [The NLRB] worked. It created the middle class. It created good jobs.[...] If you increase workers’ purchasing power, that can create a stronger, more sustainable economy.¹

This is a fundamentally macroeconomic claim. However, to our knowledge no one has used a basic macroeconomic framework to ask the question we pose above.

Workers face a basic tradeoff between the size of total income and the share of that total that they are paid.² If they choose to take a higher-than-competitive share, total income may be

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² We abstract from a potential productivity enhancing role of unions. However, Freeman and Medoff (1979, 1984), Malcomson (1983), Faith and Reid (1987) and Skovsgaard and Sena (2005) focus on the productive agency services that unions may provide.
lower because the return to investments is lower; less capital is available per worker as a result.

However, they get a larger share of that smaller total income.

We derive (from the perspective of workers) the optimal labor share in a decentralized version of the neoclassical growth model. We assume that bargaining power is sufficiently strong such that we can treat the functional distribution of income as a choice variable for labor.\(^3\) We find that the golden rule of wages implies that workers set labor share equal to the elasticity of output with respect to labor. With only two inputs to production (capital and labor) the competitive outcome is not only Pareto optimal but also delivers the highest possible long-run wage.

We also consider a framework with two types of workers: organized and unorganized. We assume that organized labor chooses its share while unorganized workers are paid their marginal product and capital receives the residual. In this case, organized labor chooses a higher than competitive labor share. For plausible parameter values, the difference is significant when production possibilities are Cobb-Douglas. Moreover, the labor share chosen is decreasing in the elasticity of substitution between labor and capital.

This sort of analysis seems especially relevant given the dramatic decrease in most OECD labor shares since 1975. (See figure 1.) This decrease has created renewed interest in the relationship between labor shares and labor market institutions, unions in particular. Bentolilla and Saint-Paul (2003) examine changes in OECD labor shares in relation to changes in bargaining power (proxied for by labor conflict rates). Berthold et al. (2002) note that labor shares in continental European countries (unlike those of the US and UK) actually evolved with

\(^3\) What is the rate of savings which maximizes the steady-state level of consumption? In 1961, Edmund Phelps termed the solution to this problem the "Golden Rule" of growth in reference to the Biblical adage to "do unto others as you would have them do unto you". The others are the future generations. Here, we ask a similar question: what is the labor income share (the functional distribution of income) which maximizes the steady-state level of wages? Coming back to the Biblical adage, in this case, the others are the future generations of workers.
a “hump”; rising in the 1970s and then falling in the 1980s. They associate this pattern with the (initially positive and then subsequently negative) effects of policies that increase both bargaining power and dismissal restrictions. Giammarrioli et al. (2002) also link the post-1980 decreases in European labor shares to changes in bargaining power and dismissal restrictions. Young and Zuleta (2011), for the US case specifically, find that industry-level union membership rates are positively associated with labor shares, but the effects are relatively small in manufacturing industries. Lastly, Bental and Dominguin (2010) introduce a model where labor’s relative bargaining power is in part a function of, on the one hand, workers’ potential to shirk subsequent to contracting and, on the other hand, the irreversibility of investment. Decreasing OECD labor shares, they argue, is consistent with increased capital mobility and improved monitoring of tasks.4

While the above studies are important theoretical and empirical contributions to understanding what may underlie the evolution of labor shares, it is also interesting to ask: in the long-run, are workers *qua* a class better off receiving a larger than competitive share. Our analysis is strictly neoclassical in the sense that we confine ourselves to “the long run[;] the land of the margin” (Solow, 1956, p. 66). Labor and capital will be fully employed at every instant; labor is supplied inelastically on competitive factor markets. There is no unemployment, but labor is essentially passive; workers are wage-takers. We then contrast the resulting steady-state to one where labor sets its own share. Evoking Phelps (1961), we essentially ask what share of the pie the good workers of Solovia would demand from their King.

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4Increased capital mobility is largely associated with globalization. Harrison (2002), Guscina (2006), and Jaumotte and Tytell (2007) all document an empirical link between increased globalization and decreased labor’s shares. Decreasing labor’s shares can also be explained by capital-using and labor-saving innovations (Zuleta (2008), Peretto and Seater (2006), and Zeira (2006)).
As far as asking such questions in a basic growth model framework, our work is most closely aligned with that of Hamada (1967), Lancaster (1973), and Kaitala and Pohjola (1990). These authors ask whether non-cooperative games between labor and capitalists can lead to Pareto optimal equilibria. Lancaster’s (1973) original conclusion using a Harrod-Domar model was that such equilibria cannot be supported. However, allowing for substitutability between labor and capital makes a difference (Solow, 1956). Incorporating a neoclassical production function, Hamada (1967) determines that Pareto optima can be supported and Kaitala and Pohjola (1990) extend the result to the case of an endogenous savings decision. We, on the other hand, ask if labor can do better than the competitive outcome in any case (Pareto optimal with the capitalists or otherwise).

This paper has the following structure. Section 2 outlines the basic model and derives the golden rule of wages. We extend that model to include both organized and unorganized labor in section 3. When we do so, organized labor will then opt for a higher-than-golden rule share. The difference is economically large for plausible parameter values. It is also larger when the elasticity of substitution between capital and labor is smaller. Section 4 then presents a multi-sector version of the model where labor in each sector independently chooses their share of that sector’s output. Despite no coordination across labor in different sectors, and despite the fact that the choice of labor share in a given sector affects the relative price of that sector’s output, the golden rule of wages still holds; in each sector labor chooses the competitive share. We briefly summarize our findings in the concluding section 5.

2. The Model
Consider a decentralized economy with one sector producing a good using capital and labor in a neoclassical production function \( Y_t = F(K_t, L_t) \).\(^5\) For simplicity we also assume that population is constant and constitutes a supply of labor that is supplied inelastically. There is no depreciation.

We assume that labor share is \( 0 < \alpha < 1 \) and, for the moment, that it is exogenous. (Below we will allow labor to dictate a specific value to maximize their long-run wage rate.) The interest rate and the wage rate are those that are consistent with workers receiving that share of total income and capital being paid the residual:

\[
(2.1) \quad r_t = (1 - \alpha) \frac{F(K_t, L_t)}{K_t} \\
(2.2) \quad w_t = \alpha \frac{F(K_t, L_t)}{L_t}.
\]

In contrast to (2.1) and (2.2), it is well known that competitive factor markets would lead to a labor share equal to labor’s production elasticity, \( \alpha_{\text{competitive}} = \frac{\partial F(K_t, L_t)}{\partial L_t} \left( \frac{1}{F(K_t, L_t)} \right) \frac{L_t}{F(K_t, L_t)} \). This competitive share will serve as the baseline for comparison.

There is an infinitely-lived representative agent who decides how much of \( Y \) to consume and how much to save at all instants. This agent can be interpreted in two ways. First, the agent may represent capitalists as a class. Second, the agent may represent the savings decisions of both capitalists and workers, as long as the latter’s savings decisions are made taking the interest rate as a given. In other words, this second interpretation makes sense if workers choose labor share (\( \alpha \)) to maximize their long-run wage rate and do not explicitly consider the effects of such

\(^5\) There are constant returns to scale in \( K \) and \( L \); first (second) derivatives are strictly positive (negative); the Inada conditions are satisfied.
a decision on the return to capital (and, therefore, their savings). This seems both empirically plausible and, importantly, if it were not the case then workers presumably would choose an $\alpha$ closer to the competitive outcome.

The lifetime utility of the representative agent is given by,

$$U = \int_0^\infty U(c_i) e^{-\rho t}$$

where $c_i$ is consumption per capita ($C_t/L_t$) and $\rho > 0$ is the agent’s subjective rate of time preference. The instantaneous utility function ($U(c_i)$) is continuous, differentiable, monotonically increasing, and concave. The representative agent solves the problem,

$$\max \int_0^\infty U(c_i) e^{-\rho t} \quad \text{subject to} \quad \dot{a}_t = w_t + r a_t - c_t,$$

where $a_t$ represents the agent’s per capita claims on physical assets. For the economy as a whole, per capita assets are equal to the per capita capital stock ($a_t = k_t$). Solving this problem leads to the standard Euler equation,

$$\frac{U'(c_i)}{U''(c_i)} \dot{c}_t = r_t - \rho.$$

If there is a steady state then,

$$(1 - \alpha) \frac{F(K, L)}{K} = \rho,$$

and combining (2.5) with equation (2.2) leads to,

$$w = \frac{\alpha}{1 - \alpha} \frac{\rho K}{L},$$

where dropping “$t$” subscripts indicates a steady-state value. Given the assumptions we made about the production function and given the labor share ($\alpha$), the steady-state is unique and stable.
Now assume that while the representative agent takes \( \alpha \) as exogenously given, labor share is actually collectively chosen by workers. If we assume that workers choose \( \alpha \) such that their long-run (steady-state) wage rate is maximized, we arrive at the following proposition.

**Proposition 2.1:** If the workers can choose \( \alpha \) in order to maximize their long run wage rate then 
\[
\alpha = 1 - \beta ,
\]
where \( (1 - \beta) \) is the elasticity of output with respect to labor (and \( \beta = F_k(K, L) \frac{K}{Y} \) is the elasticity of output with respect to capital).

**Proof**

Applying the implicit function theorem to (2.5) we find

\[
\frac{\partial K}{\partial \alpha} = \frac{K}{(1 - \alpha) \left( \frac{F_k(K, L)}{F(K, L)} \frac{K}{K - 1} \right)} = -\frac{K}{(1 - \alpha)(1 - \beta)}.
\]

Workers solve the following problem:

\[
\max_{\alpha} \{w\} \text{ subject to (2.7)} \quad \text{where } w \text{ is defined by the time-invariant version of (2.2).}
\]

Taking logs of (2.2) and then the derivative with respect to \( \alpha \) yields,

\[
\frac{\partial \log w}{\partial \alpha} = \frac{1}{1 - \alpha} + \frac{1}{\alpha} + \frac{1}{K} \frac{\partial K}{\partial \alpha} = 0
\]

Substituting (2.7) into the derivative above and setting equal to zero leads to,

\[
\frac{\partial \log w}{\partial \alpha} = \frac{1}{(1 - \alpha) \alpha} - \frac{1}{(1 - \alpha)(1 - \beta)} = 0
\]

which will be true when \( \alpha = (1 - \beta) \). QED.

It is well-known that the decentralized solution to the neoclassical growth model is Pareto optimal (Barro and Sala-i-Martin (1995, pp. 98-99)). Intuitively, up until the competitive capital
per worker, everyone can be made better off by additional investment. Relative to that Pareto optimum, workers cannot possibly make themselves better off without making someone else (i.e., capitalists) worse off. This evokes a response from the capitalists in the form of a lower rate of investment. From (2.7) we see that, \( \frac{\partial K}{\partial \alpha} < 0 \) and \( \frac{\partial^2 K}{\partial \alpha^2} > 0 \) for all \( \alpha \) and \( \beta \) between 0 and 1.

Choosing a larger labor share is costly in terms of capital available per worker, and increasingly so. Proposition 2.1 establishes that, due to this response on the part of capitalists, an attempt by labor to improve their lot at the expense of capital will not be successful. Both labor and capital will be left worse off.

3. Organized Versus Unorganized Labor

In the neoclassical growth model there is no long-run conflict between capital and labor in terms of their relative income shares. The labor share preferred by workers in terms of maximizing their long-run wage rate is equal to the competitive share. This conclusion is based on the assumption of there being only two productive factors: capital and labor. In this section we extend the analysis to consider, along with capital, two types of labor.

Berthold et al. (2002, p. 437) reports that, from 1974 to 1975, “it can be tentatively concluded that across OECD countries unemployment rose more where cumulative changes of the labor share were also large.” This would be consistent with increased bargaining power and dismissal restrictions benefiting some workers at the expense of others. Our model does not have unemployment, but we explore the essential story by assuming two classes of workers – one with
the power to dict ate their income share (organized labor) and the other that are wage-takers (unorganized labor).6

Consider the same decentralized economy except that now there are the two labor types: organized \((L_O)\) and unorganized \((L_N)\). For simplicity, both labor types are assumed to be equally productive and inelastically-supplied.7 There are still constant returns to scale, now in the three inputs \((K, L_O, \text{and } L_N)\). There are diminishing returns to each factor holding the other two fixed.

Organized workers choose their labor share \((0 < \alpha < 1)\) to maximize their long-term wage \((w_O)\). Unorganized workers, on the other hand, are paid a wage \((w_N)\) equal to their marginal product and their income share is consistent with this wage. The residual income accrues to capital and this determines the rental rate \((r)\). The assumptions lead to the following steady-state relationships:

\[
(3.1) \quad w_O = \alpha \frac{F(K, L_O, L_N)}{L_O};
\]

\[
(3.2) \quad w_N = \frac{\partial F(K, L_O, L_N)}{\partial L_N} \quad \text{and} \quad \gamma = \frac{w_N L_N}{F(K, L_O, L_N)};
\]

\[
(3.3) \quad r = (1 - \alpha - \gamma) \frac{F(K, L)}{K};
\]

\[
(3.5) \quad (1 - \alpha - \gamma) \frac{F(K, L_O, L_N)}{K} = \rho.
\]

Combining (3.1) with (3.5) yields,

\[
(3.6) \quad w_O = \left(\frac{\alpha}{1 - \alpha - \gamma}\right) \left(\frac{K}{L_O}\right) \rho.
\]

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6 While we will assume distinct types of workers below – one of which has a direct say in the functional distribution of income; one which does not – this can be viewed as an extreme case of the “wage compression” model of unions where the median worker is unskilled and the outcome of the bargaining process is a capturing of some marginal production associated with skilled workers (e.g., Acemoglu et al. (2001) and Hornstein et al. (2005)).

7 Assuming that one labor type to be relatively productive would not affect our results qualitatively.
From this point we can move directly to a proposition analogous to proposition 2.1 above.

Proposition 3.1: If organized workers choose \( \alpha \) in order to maximize \( w_o \) then

\[
\alpha = \frac{(2 - \beta)(1 - \gamma) - (1 - \gamma) \beta^2}{2} - \left( \frac{4 \gamma}{(1 - \gamma) \partial K} \right) \frac{K}{Y}
\]

where \( \beta = F_k(K, L) \frac{K}{Y} \) is the elasticity of output with respect to capital.

Proof

Applying the implicit function theorem to (3.5) we find that,

\[
\frac{\partial K}{\partial \alpha} = -\frac{K}{(1 - \alpha - \gamma)(1 - \beta) + \frac{4 \gamma}{\partial K} K}
\]

Organized workers solve the following problem: \( \max_{\alpha} \left[ w_o \right] \) subject to the above and the fact that \( w \) is defined by (3.6). Taking logs of (3.6) and then the derivative with respect to \( \alpha \) yields,

\[
\frac{\partial \log w_o}{\partial \alpha} = \frac{1}{1 - \alpha - \gamma} + \frac{1}{\alpha} \frac{\partial K}{\partial \alpha} = 0
\]

Substituting (3.7) into the derivative above and setting equal to zero leads to

\[
\alpha = (1 - \gamma) \left[ (1 - \beta) + \frac{\partial \gamma}{\partial K} \frac{1}{(1 - \alpha - \gamma) K} \right]
\]

From (3.9) we can state the following:

(a) if \( \frac{\partial \gamma}{\partial K} > 0 \) then \( \alpha > (1 - \gamma)(1 - \beta) \);

(b) if \( \frac{\partial \gamma}{\partial K} < 0 \) then \( \alpha < (1 - \gamma)(1 - \beta) \);
(c) if \( \frac{\partial \gamma}{\partial K} = 0 \) then \( \alpha = (1 - \gamma)(1 - \beta) \).

From (3.9) we can also solve for \( \alpha \):

\[
(3.10) \quad \alpha = \frac{(2 - \beta)(1 - \gamma) \pm (1 - \gamma)\beta}{1} \sqrt{\beta^2 - \left(\frac{4 \frac{\partial \gamma}{\partial K}}{1 - \gamma}\right) K}.
\]

From equation (3.10) we can see that \( \alpha = \frac{(2 - \beta)(1 - \gamma) \pm (1 - \gamma)\beta}{2} \) if \( \frac{\partial \gamma}{\partial K} = 0 \) and the following are true:

(i) if \( \frac{\partial \gamma}{\partial K} > 0 \) then \( \alpha < (1 - \gamma) \) is associated with the positive root and

\[ \alpha > (1 - \beta)(1 - \gamma) \] is associated with the negative root;

(ii) if \( \frac{\partial \gamma}{\partial K} < 0 \) then \( \alpha > (1 - \gamma) \) is associated with the positive root and

\[ \alpha < (1 - \beta)(1 - \gamma) \] is associated with the negative root;

(iii) if \( \frac{\partial \gamma}{\partial K} = 0 \) then \( \alpha = (1 - \gamma) \) is associated with the positive root and

\[ \alpha = (1 - \beta)(1 - \gamma) \] is associated with the negative root.

If we compare (i), (ii), and (iii) with (a), (b), and (c) above, we see that they are consistent only for the negative roots. Also, for (ii) and (iii) the positive roots would imply, respectively, that capital’s share is negative or zero. We conclude that,

\[
(3.11) \quad \alpha = \frac{(2 - \beta)(1 - \gamma) - (1 - \gamma)\beta}{2} \sqrt{\beta^2 - \left(\frac{4 \frac{\partial \gamma}{\partial K}}{1 - \gamma}\right) K}.
\]

QED.
According to (3.11), organized labor may choose an income share different from the competitive share. In particular, the competitive share is,

\[(3.12) \quad \alpha = (1 - \beta - \gamma).\]

The point of comparison comes from the Cobb-Douglas case where the elasticity of substitution between factors is unity:

\[(3.13) \quad \alpha = (1 - \beta)(1 - \gamma) = (1 - \beta - \gamma) + \beta \gamma.\]

Note that if \(\gamma = 0\) (which is the one-labor-type case of section 2) then \(\alpha = (1 - \beta)\). However, for positive values of the share/elasticity, \(\gamma\), organized labor share will be greater than the competitive share by \(\beta \gamma\). This may be a sizeable difference. For example, if \(\beta = \gamma = 0.30\) then organized labor will choose a share of 0.49 rather than the competitive share of 0.40.

**Table 1** presents, for the Cobb-Douglas case and for \(\beta\) and \(\gamma\) values of 0.10, 0.30, and 0.50, (i) labor shares chosen by organized labor, (ii) corresponding competitive labor shares, and (iii) the ratios of the former to the latter. In general, the lower the capital elasticity (\(\beta\)) the higher organized labor’s preferred share. Also, for \(\beta = 0.30\) (perhaps the most oft-assumed value for this parameter) organized labor’s preferred share is at least five percent higher than the competitive share for all reported \(\gamma\) values.

The intuition as to why organized labor may prefer a higher-than-competitive income share when there is both unorganized labor and capital (as opposed to only capital) is straightforward. In the model with only one type of labor, starting from the Pareto optimum workers can only attempt to gain higher wages at the expense of capital. Doing so causes capitalists to accumulate less capital per worker. On the other hand, unorganized labor is inelastically supplied. Given that all of the unorganized labor input will be employed, what is
key to organized labor’s choice of $\alpha$ is how the marginal product of unorganized labor (and therefore its own share) changes in response to having less capital \( \frac{\partial \gamma}{\partial K} \). In the Cobb-Douglas case, factor elasticities (including $\gamma$) do no change when the capital stock decreases. The decrease in total income from a reduction capital per worker is spread across organized and unorganized workers. For this reason, the income share that organized workers choose is higher than the competitive one.

In terms of the above derivative, the empirically relevant case is probably not a Cobb-Douglas but rather a less-than-unity elasticity of substitution.\(^8\) For example, recent studies of the US by Antràs (2004), Chirinko et al. (2007), Klump et al. (2007), and Young (2011) all report a substitution elasticity between labor and capital significantly below unity \( \frac{\partial \gamma}{\partial K} > 0 \). If this is the case, then organized labor will choose $\alpha$ greater than both the competitive share and that which would be chosen given Cobb-Douglas technology. The difference will be decreasing in both $\gamma$ and \( \frac{\partial \gamma}{\partial K} \).

4. **Multiple Sectors**

Rather than assuming a single sector with two labor types, it may be more realistic to think of multiple sectors where labor in each sector bargains over the output of that particular sector. We demonstrate in this section that when all labor is organized and capital is the only other input, then the golden rule of wages holds for each sector. We do so under the assumption that labor does not coordinate across sectors. This is the interesting case because, alternatively, if labor in

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\(^8\) This is based on assuming a constant elasticity of substitution production function where the elasticity is the same across capital, organized labor, and unorganized labor.
each sector does coordinate, then aggregate labor’s problem obviously collapses to that which is described in section 2 above.

Assume an economy with $N$ sectors producing consumption goods, $i = 1, ..., N$. Each good is produced using a constant returns technology using capital ($K_i$) and labor ($L_i$). Labor in each sector is organized and chooses their share of that sector’s production.

The representative consumer maximizes lifetime utility similar to above but this time with a consumption aggregate over $N$ goods,

\[ c = \left[ \prod_i (c_i)^\gamma \right], \]

where $0 < \gamma < 1$. The budget constraint is the same as (2.3) only now,

\[ a = \sum_i a_i \quad \text{and} \quad a_i = k_i, \]

for all $i = 1, ..., N$. Capital is assumed to be fully mobile such that,

\[ r = r_i = (1 - \alpha_i) \frac{f(k_i)}{k_i} p_i, \]

for all $i = 1, ..., N$. In (4.3), $p_i$ is the relative price of good $i$. There is some sector, $j$, for which we will assume that $p_j = 1$.

In a steady-state,

\[ (1 - \alpha_i) \frac{f_i(k_i)}{k_i} p_i = \rho; \]

\[ w_i = \frac{1}{\beta} \frac{\alpha_i}{1 - \alpha_i} k_i \]

and

---

The specific form of the consumption aggregate is not important, provided it is consistent with an Euler equation of the standard form ((4.4) below).
\[
\frac{p_i}{p_j} = \frac{f(k_j)}{f(k_i)} \quad \text{or} \quad p_i = \frac{f(k_j)}{f(k_i)},
\]

where, again, good \( j \) is assumed to be the numeraire good.

If labor shares are choice variables, then

\[
w_i = \alpha_i p_i f(k_i) .
\]

Note that, if labor does not coordinate across sectors, then labor in sector \( i \) faces a problem different than that described in section 2 because their choice affects the relative price of their sector’s output (\( p_i \)). From (4.4) we can determine that, in the steady-state,

\[
p_j = \frac{1}{\beta (1 - \alpha_j)} \frac{k_j}{y_j},
\]

and therefore,

\[
\frac{\partial p_i}{\partial k_i} = \frac{1}{\beta (1 - \alpha_j)} \frac{y_i - f'(k_i) k_i}{y_i (k_i)} = \frac{1}{\beta} \frac{y_i - f'(k_i) k_i}{y_i (k_i)} (1 - \beta); 
\]

\[
\frac{\partial p_i}{\partial \alpha_i} = p_i \frac{1}{(1 - \alpha_j)} .
\]

We can now state the main result of this section.

**Proposition 4.1**: If the workers of sector \( i \) can choose \( \alpha_i \) in order to maximize the long run wage rate then \( \alpha_i = 1 - \beta_i \), where \( (1 - \beta_i) \) is the elasticity of output with respect to labor (and

\[
\beta_i = \frac{F_{ik}(K_i, L_i) - K_i}{F_i(K_i, L_i)} \quad \text{is the elasticity of output with respect to capital}.
\]

**Proof**

Applying the implicit function theorem to (4.6) we find that,
\[
\frac{\partial k_i}{\partial \alpha_i} = - \frac{k_i \left( p_i - (1 - \alpha_i) \frac{\partial p_i}{\partial \alpha_i} \right)}{(1 - \alpha_i) \left[ (1 - \beta_i) p_i - \frac{\partial p_i}{\partial k_i} k_i \right]}
\]

Taking logs of (4.5) and then the derivative with respect to \( \alpha_i \) results in,

\[
\frac{\partial \log w_i}{\partial \alpha_i} = \frac{1}{1 - \alpha_i} + \frac{1}{\alpha_i} + \frac{\partial k_i}{\partial \alpha_i}.
\]

Substituting (4.10) into (4.11) and setting that derivative equal to zero:

\[
\frac{\partial \log w_i}{\partial \alpha_i} = \frac{1}{1 - \alpha_i} - \frac{\left( p_i - (1 - \alpha_i) \frac{\partial p_i}{\partial \alpha_i} \right)}{(1 - \alpha_i) \left[ (1 - \beta_i) p_i - \frac{\partial p_i}{\partial k_i} k_i \right]} = 0.
\]

Inspection of (4.12) tells us that if \( \alpha_i = (1 - \beta_i) \) then the equality will hold only if

\[
(1 - \alpha_i) \alpha_i \frac{\partial p_i}{\partial \alpha_i} = \frac{\partial p_i}{\partial k_i} k_i. \text{ But from (4.8) and (4.9) we already know that if } \alpha_i = (1 - \beta_i) \text{ then}
\]

\[
(1 - \alpha_i) \alpha_i \frac{\partial p_i}{\partial \alpha_i} = \frac{\partial p_i}{\partial k_i} k_i.
\]

\textit{QED.}

5. Conclusions

We consider a model where the share of income paid out to labor depends only on the bargaining power of workers and assume that workers set the labor share in such a way that long run (permanent) wages are maximized. In this framework, if the labor share increases relative to the competitive share, workers capture a larger share of a smaller total income in the steady-state. This is because the incentives to invest are lower and the steady-state capital to labor ratio is
lower. In other words, there is a tradeoff, having a relatively large slice of the income pie and having a larger pie in the first place.

What is the labor share which maximizes the steady state wage level given this tradeoff? We derive this labor share in a decentralized version of the neoclassical growth model where the functional distribution of income is determined by the bargaining power of workers. We find that the “Golden Rule” labor share is equal to the elasticity of output with respect to labor. This is precisely what would obtain under the assumption of competitive factor markets.

We also consider a framework with organized and unorganized workers. In this case, for plausible parameter values organized labor chooses a higher than competitive labor share and the difference is economically significant for plausible parameter values. Furthermore, relative to the Cobb-Douglas case, organized labor chooses a higher share for the empirically relevant case of an elasticity of substitution less than unity.

Compared to the model with only capital and organized labor, we see that organized labor only gains from a higher-than-competitive share at the expense of unorganized labor. The intuition is straightforward: whereas an attempt to gain at the expense of capital lowers the return to investment and results in less capital and total income, unorganized labor continues to be inelastically supplied when its share is decreased. Perfectly inelastic unorganized labor supply is an extreme assumption. However, we believe that a model where capitalists are relatively (to unorganized laborers) willing and able to react to changes in returns captures a plausible feature of real economies.

We also analyze a multi-sector version of the model where labor in each sector chooses their share of that sector’s output. Despite assuming no coordination across labor in different sectors, and despite the fact that, in this setting, the choice of a sectoral labor share affects the
relative price of that sector’s output, the golden rule of wages still holds. Each sector’s labor can do no better than to choose the competitive labor share.
References


Review of Economic Studies 34, 295-299.


Table 1 – Organized labor share for various factor elasticities and Cobb-Douglas technology

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<thead>
<tr>
<th>Capital elasticity</th>
<th>Unorganized labor elasticity</th>
<th>Organized labor share (chosen)</th>
<th>Organized labor share (competitive)</th>
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Figure 1 – Change in OECD Labor Shares, 1975-2000

Source: United Nations Data Set. The data was taken from Finnoff and Jayadeff (2006). They used the United Nations dataset to construct the data on labor shares.